Vision of Chapter 6

"Sausage waves and oscillations in the solar atmosphere"

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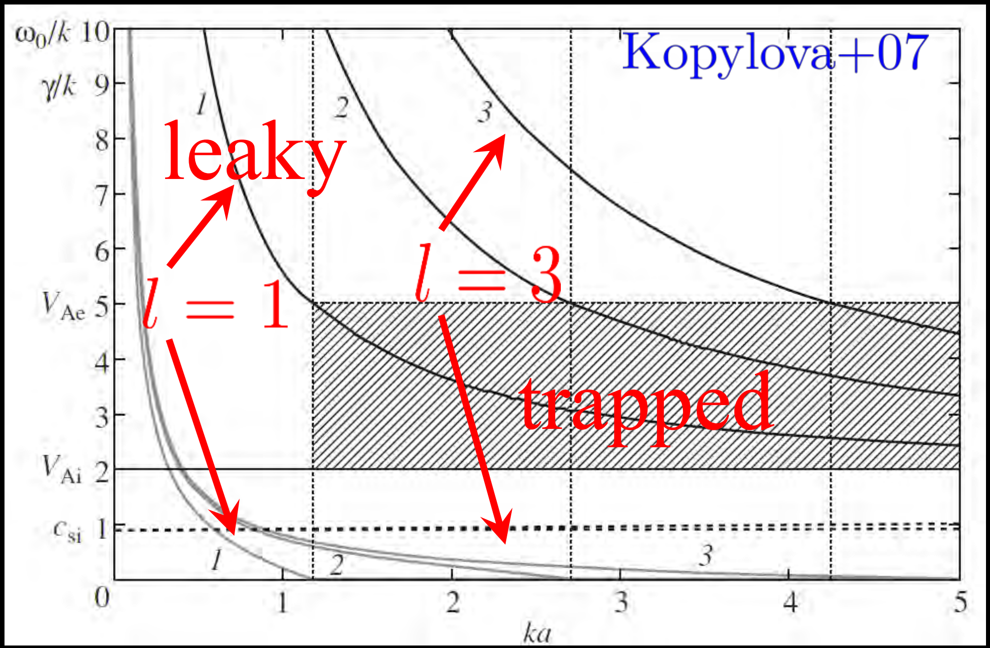
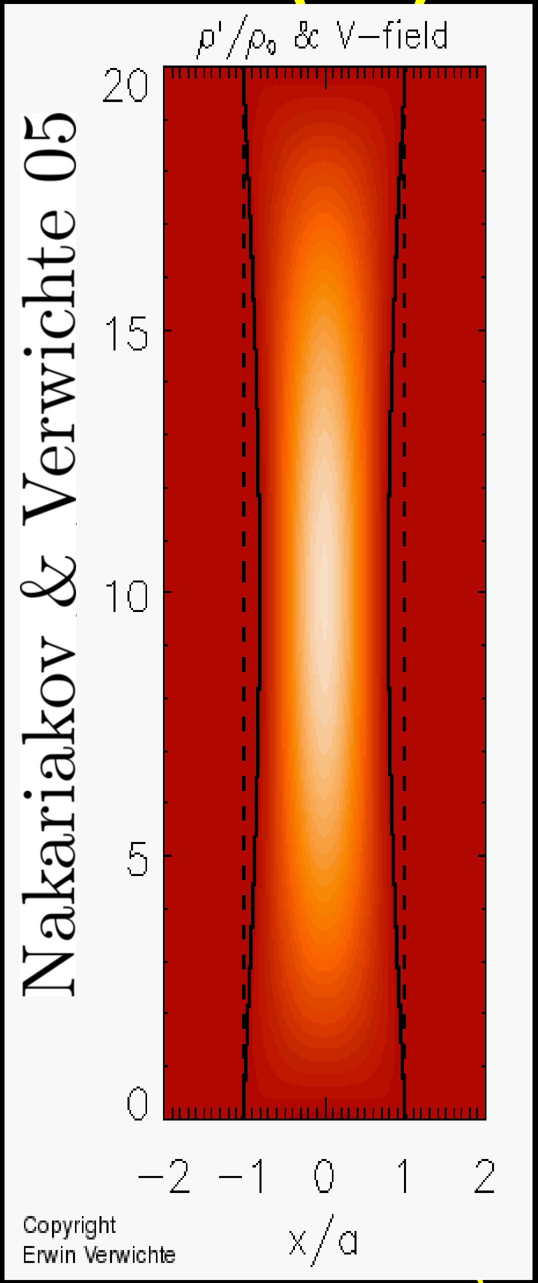
Materials to cover

- Not to cover coronal slow sausage waves (essentially field-guided acoustic modes, covered by Tongjiang & Dipankar)
- Nomenclature biased towards coronal fast sausage waves
  - Nomenclature from Edwin & Roberts 83
  - Connection to observations & illustrative seismological practice
  - Words of caution
- Sausage waves/oscillations in the solar corona
  - Observational signatures from forward modeling efforts
  - Theoretical advances
  - seismology
- Sausage waves/oscillations in the lower solar atmosphere
  - Observational signatures and potential seismology
  - Heating aspects: energy carrying capabilities and damping/dissipation

Nomenclature from Edwin & Roberts (83)



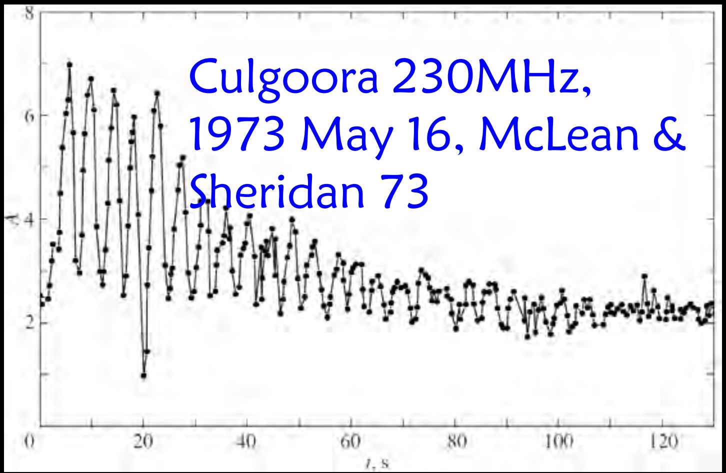


- Equilibrium: straight, field-aligned, static, circular, trans. step, long. uniform, …
- Sausage
  - perturbation parity, strong dispersion, strong compression
  - axial/transverse fundamental + harmonics
  - cutoff axial wavenumber (trapped, leaky) → period~R/v_Ai~seconds
  - NO resonant coupling to the Alfven continuum

Connection to observations

- Primarily, rapid QPPs (reviews Nakariakov & Melnikov 09, Van Doorsselaere+16, McLaughlin +18, Ivan?)
  - spatially unresolved (compiled in Aschwanden+04, also Van Doorsselaere+11, …)
  - spatially resolved
    - ✓ Radio: Nakariakov+03, Melnikov+05, Kolotkov+15, 18, Nakariakov+18 microflare,…
    - ✓ (E) UV: Su+12, Tian+16, Dennis+17?
- other than QPPs?

Illustrative seismology (Roberts+83, 84)



$$P \approx \frac{2.6R}{v_{\rm Ai}} \qquad \tau/P \approx \frac{\rho_{\rm i}/\rho_{\rm e}}{\pi^2}$$

$$P \sim 4.3 \text{ sec}, \quad \tau/P \sim 10$$

$$R/v_{\rm Ai} \sim 1.6 \text{ sec}, \quad \rho_{\rm i}/\rho_{\rm e} \sim 100$$

- Standing modes (also Roberts 19, Chen+15, …)
- impulsively generated wave trains (also Edwin & Roberts+86, Roberts 00, 19; Nakariakov+04, …)

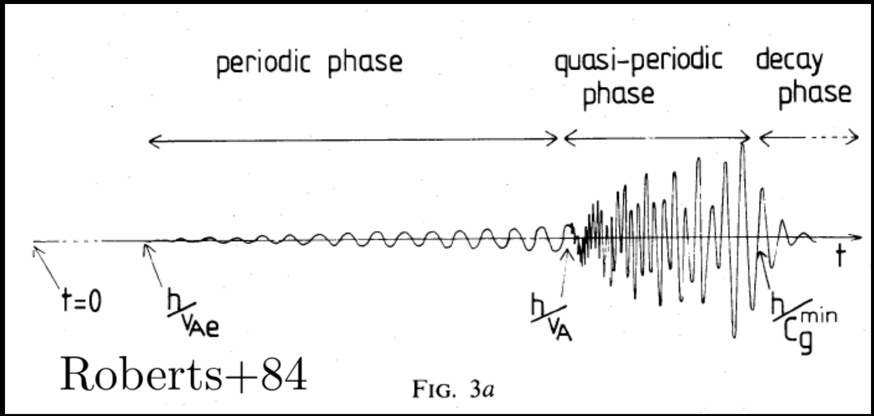
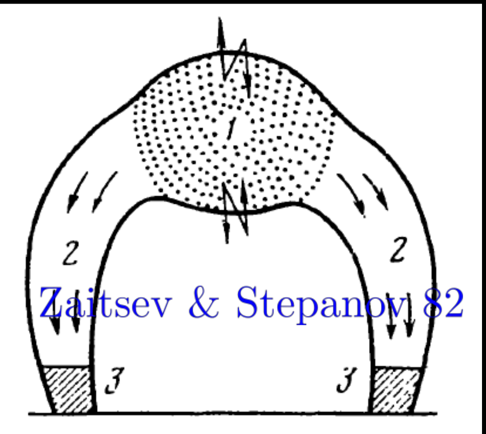


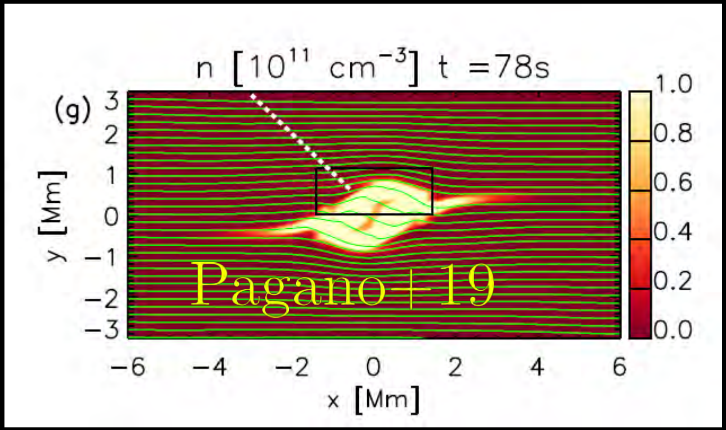
FIG. 3a

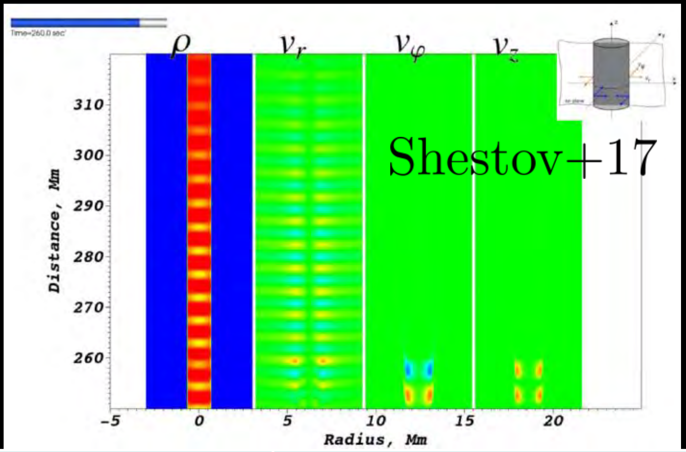
useful for seismology (in principle)

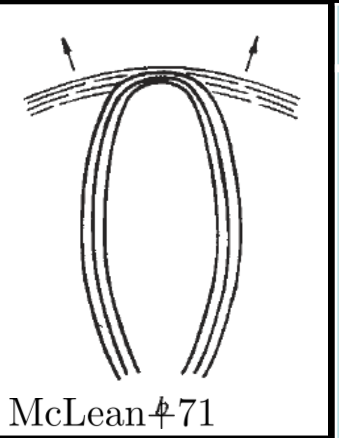
- quasi-period
- duration of quasi-periodic & Airy phases
- …

Caveat 1: Generation



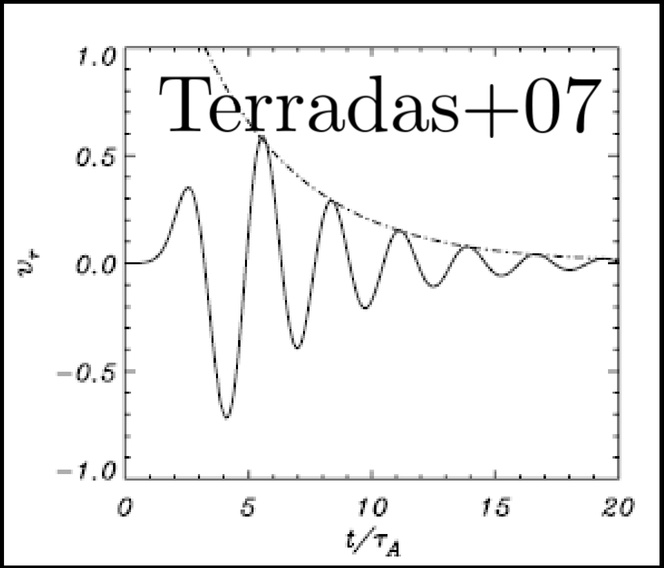


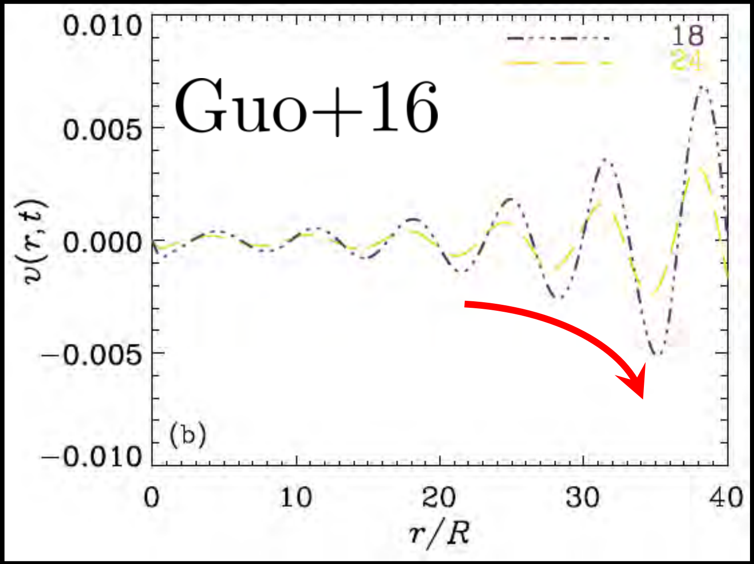




	Driver	Applicable to
Internal	eigen-mode amplification due to resonance with bouncing fast ptcls (Meerson+78)	flare loops
	impulsive energy release (Zaitsev & Stepanov 82)	flare loops
	collision of (say, coronal rain) clumps (Antolin+18, Pagano+19)	(AR) loops (exclusively so?)
	driven nonlinearly by torsional Alfven (Shestov+17)	AR loops
External	interaction between flare-driven shock and looptop (McLean 71)	flare loops
	photospheric converging flow (Berghmans+96)	AR loops

Caveat 2: physical reality of leaky modes





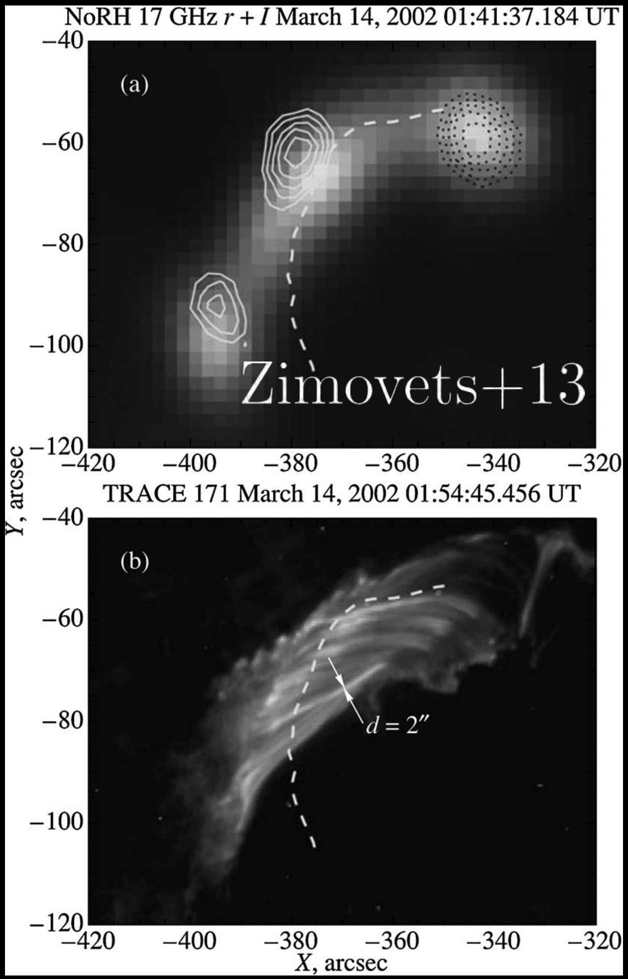
- Apparently counter-intuitive, must be understood from an IVP perspective (Cally 86), indeed confirmed (but only for simple eq.)
- Interference of improper continuum modes, period due to enhanced spectral measure, damping due to weakening of constructive inference (Andries & Goossens 07)

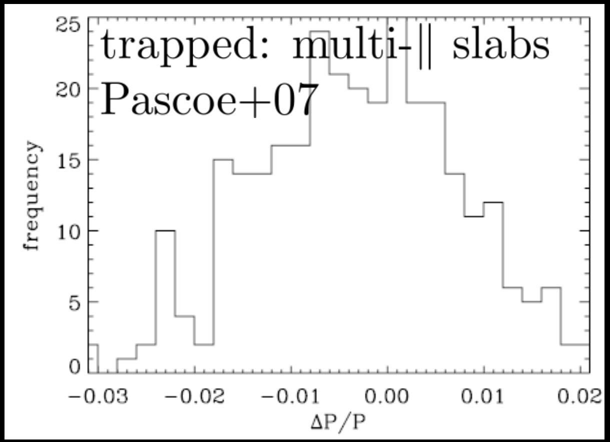
Oliver+15

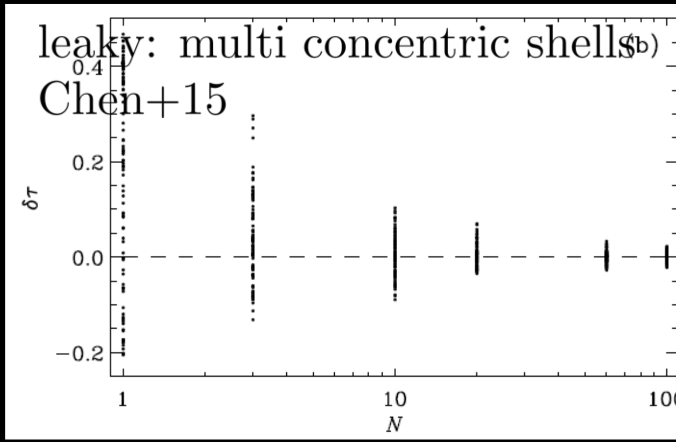
$$\tilde{\xi}_r(t, r, k) = \sum_{j=1}^{N} \left[A_j^+(k) e^{-i\omega_j(k)t} + A_j^-(k) e^{i\omega_j(k)t} \right] \hat{\xi}_j(r, k) + \int_{|k|v_{Ae}}^{\infty} \left[A_\omega^+(k) e^{-i\omega t} + A_\omega^-(k) e^{i\omega t} \right] \hat{\xi}_\omega(r, k) d\omega$$

transverse & axial extent of initial pert. matter! Discrete leaky eigen-modes not guaranteed to be seen in system evolution!

Caveat 3: multi-stranded vs. monolithic

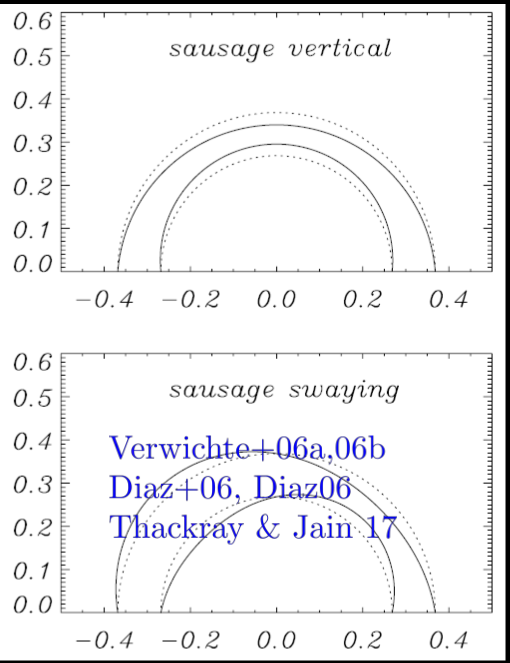


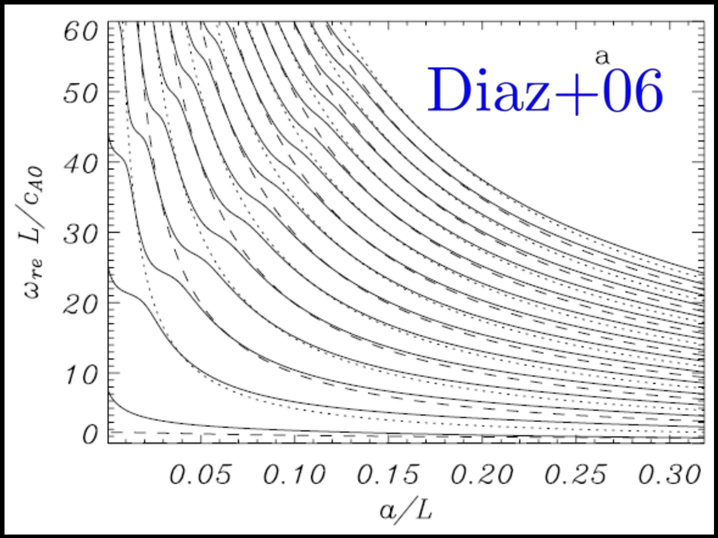


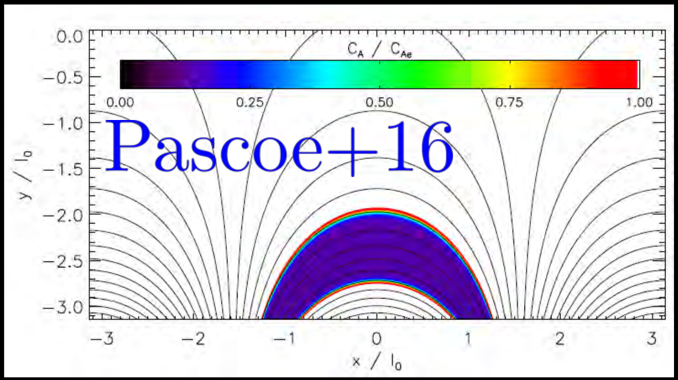


- a radio "loop" = multi EUV loops
- fine-structuring as concentric shells, neither period (Pascoe+07) nor damping time (Chen+15) affected if number of shells > 10
- Sausage-like modes expected, but NOT shown, to exist with further loss of rotational symmetry

Caveat 4: Curvature







- Transverse structuring in Alfven speed (frequency) inevitable in general. In general, NO trapped modes, regardless of R/L. (already reviewed by Van Doorsselaere+09)
- Distinction between curved & straight slabs marginal for thick+dense slabs+suitable spatial distribution of external Alfven speed (Pascoe+16)
- Behavior for thin curved slabs very different from straight case: period can be ~ longitudinal Alfven time!
- So far only curved slabs are examined

Coronal standing modes: forward modeling

- Heuristic discussions already in Nakariakov & Melnikov 09
- Recent forward modeling, exclusively for simple equilibrium
  - Non-thermal emissions
    - ✓ Microwave (1-100 GHz) from Gyro-Synchrotron (Reznikova+14, 15, Kuznetsov+15), via the fast GS code (Fleishman & Kuznetsov 10)
    - ✓ other passbands (Hard X-ray, and else)?
  - Thermal emissions
    - ✓ (E)UV (Antolin+13, Shi+19a, 19b, 19c; also Cooper+03a, 03b; Gruszecki+12), exclusively collisionally excited + spontaneous emission
    - ✓ other passbands (Soft X-ray and else?)/Radiative excitation (tall loops)?
  - periodicity, damping time, phase-difference, … depend on
    - ✓ equilibrium parameters (density, temperature, B strength, …)
    - ✓ atomic physics (e.g., non-equilibrium ionization) & plasma effects (e.g., Razin suppression)
    - ✓ instrumental parameters (spatial, temporal, spectral resolution)

Coronal sausage waves: eigen-modes

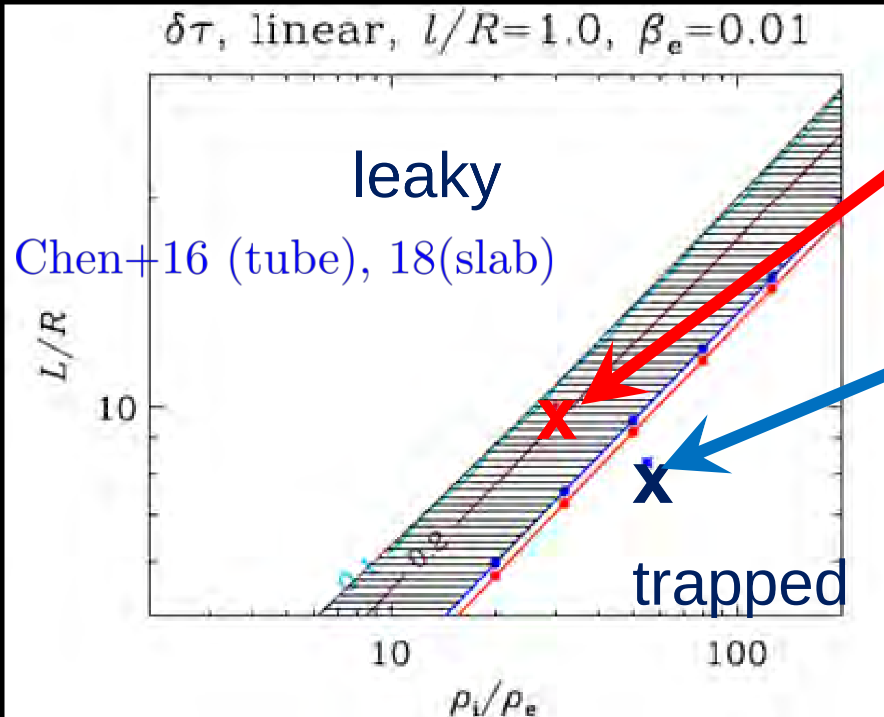
Slab Cylindrical

						inhomogeneity in v_{char}			
Reference Set	Loop Curvature?	$\vec{B}_0 \parallel$ axis?	circular cross-section?	axial flow?	azimuthal flow?	transverse step?	longitudinally uniform	temporally stationary?	gravity?
1.	N	Y	Y	N	N	Y	Y	Y	N
2.	Y		Y/N			Y/N	Y/N		
3.				Y		Y/N			
4.			N						
5.						Y/N	N		
6.						N			
7.		N				Y/N			

1. **Base model**: Edwin & Roberts 82, Edwin & Roberts 83; Rosenberg 70, Zaitsev & Stepanov 75, Meerson+78, Spruit 82, Cally 86, Kopylova+07, Vasheghani Farahani+14, Bahari 18
2. **Curved loops**: Smith+97, Verwichte+06a, 06b, Diaz+06, Diaz 06, Pascoe & Nakariakov16, Thakray & Jain 17 (also Hindman & Jain 15)
3. **Axial flow**: Li+13, Chen+14; Li+14, Yu+16a
4. **Noncircular cross-section**: elliptical – Erdelyi & Morton 09 (also Ruderman 03, Morton & Ruderman 11)
5. **Axially nonuniform**: density - Cally & Xiong 18; magnetic field – Pascoe+09
6. **Transversely continuous**: Lopin & Nagorny 15, Yu+15, Li+18 Edwin & Roberts 88; Nakariakov+12; Chen+15, 16, 18; Guo+16; Yu+16b, 17; Lopin & Nagorny 14, 15,19
7. **Twist**: Erdelyi & Fedun 07; Khongorova+12; Giagkiozis+15, 16; Lopin & Nagorny 19

- Note: eigenmodes not guaranteed to be physically relevant, i.e., show up in system evolution

Influence of plasma beta



NoRH (Kolotkov+15)

NoRH (Nakariakov+03);

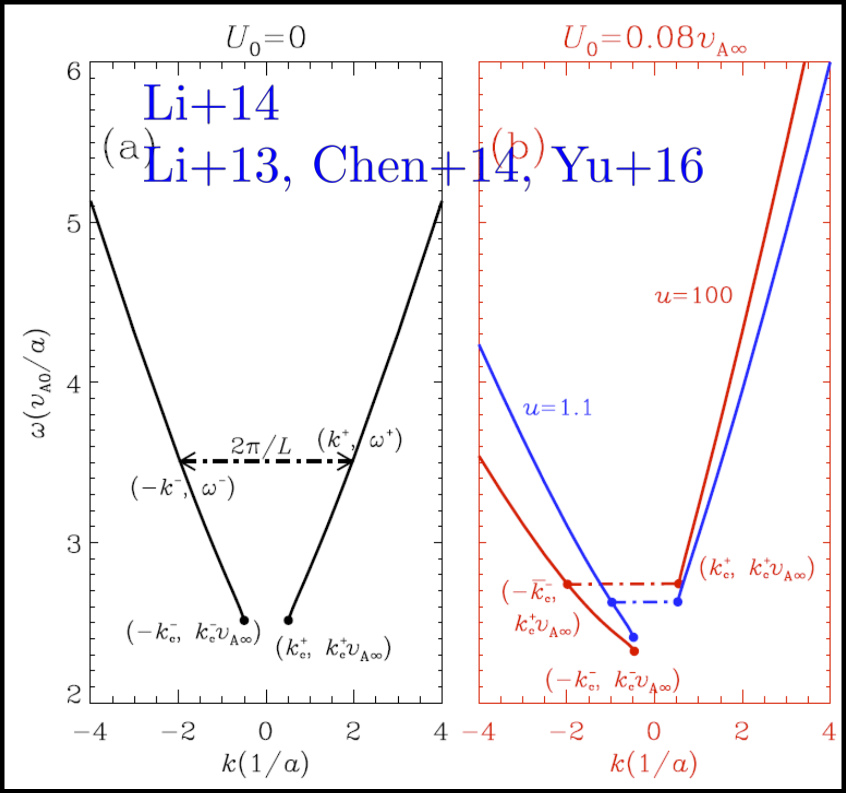
IRIS (Tian+ 16)

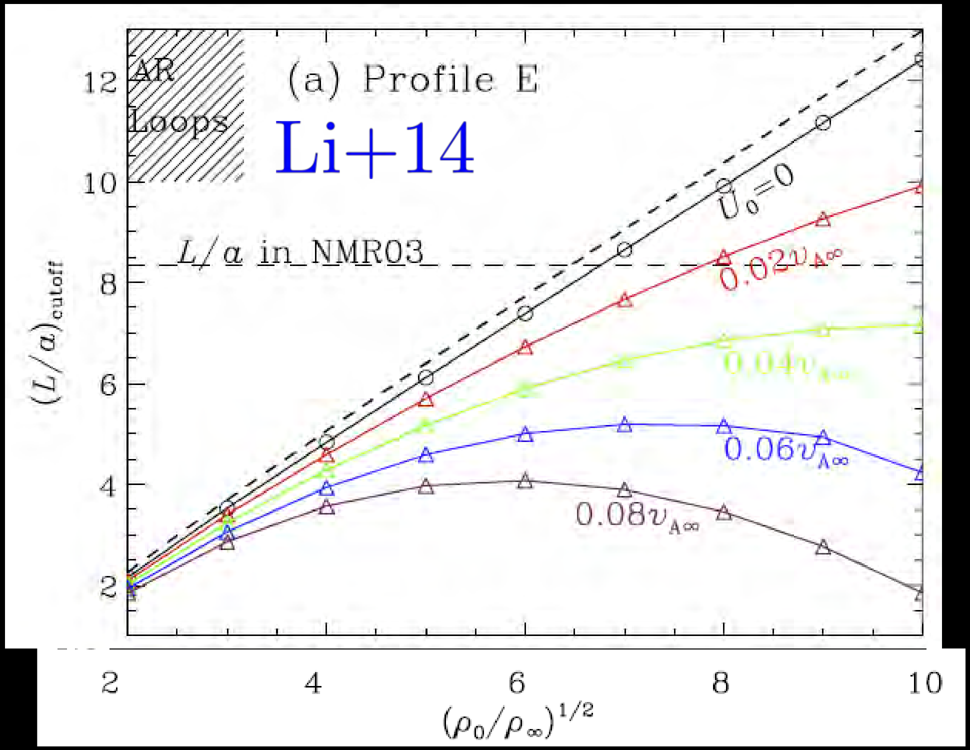
$$\frac{\omega R}{v_{\rm fi}} = \mathcal{H}\left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\rm i}}{\rho_{\rm e}}, \beta_{\rm i}, \beta_{\rm e}\right)$$

$$\delta\tau = \max\left|\frac{\tau^{\beta\neq 0}(\beta_{\rm i} \in [0, 1])}{\tau^{\rm cold}} - 1\right|$$

- P in $R/v_{\rm fi}$ not sensitive to internal beta → cold MHD results can be used to invert period, but the derived $v_{\rm Ai}$ actually means $v_{\rm fi}$
- Cold MHD results can be used to invert damping time if mode is deep in the leaky regime
- Done only for straight "loops"

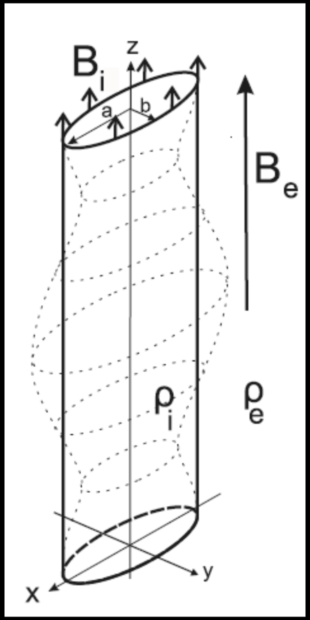
Axial flow

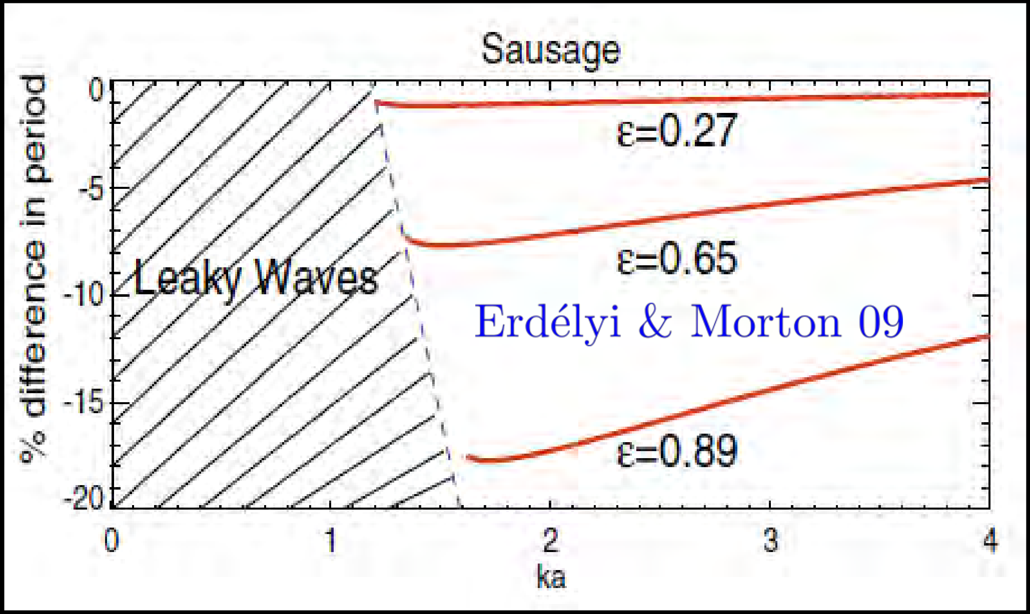




- breaks forward & backward symmetry
- even mild axial flows can significantly reduce the parameter space where trapped modes are allowed
- Note: a strict definition of standing modes

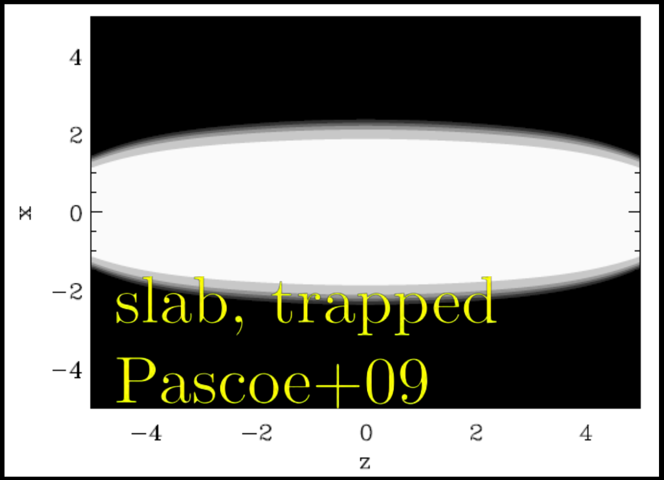
elliptic cross-section

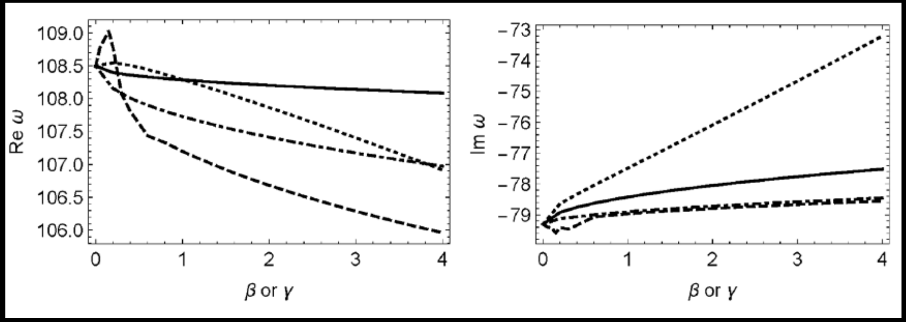


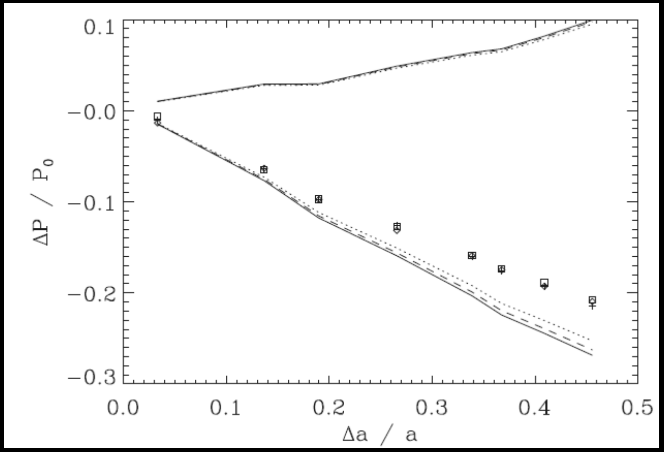


- breaks rotational symmetry, but overall similar to circular cases
  - collective "breathing"-like motions allowed, despite all even modes are coupled
  - distinction between trapped & leaky
- cutoff axial wavenumber affected by eccentricity

Axial stratification in density or B

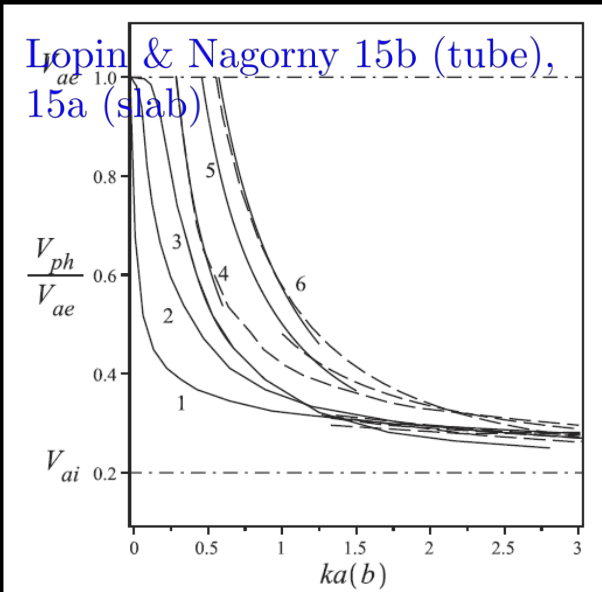


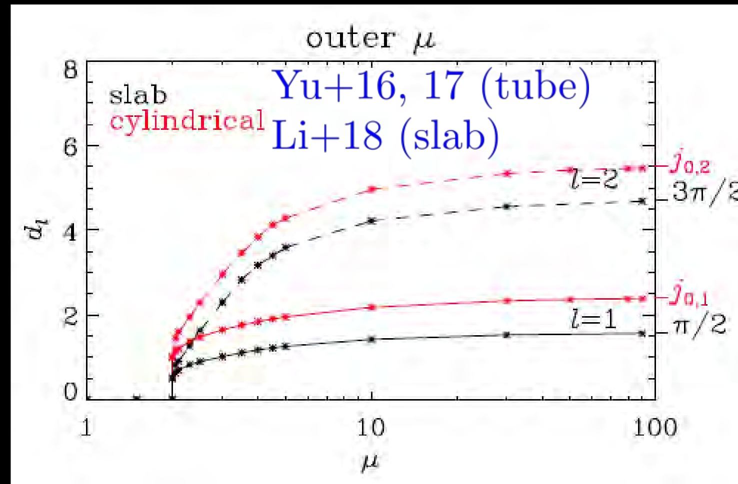




- all axial modes are coupled
- periods & damping times of axial fundamental only slightly different from unstratified case (note: this is true in Pascoe+09 when P expected with smallest half-width). Axial harmonics more affected

Transverse density structuring: step → continuous





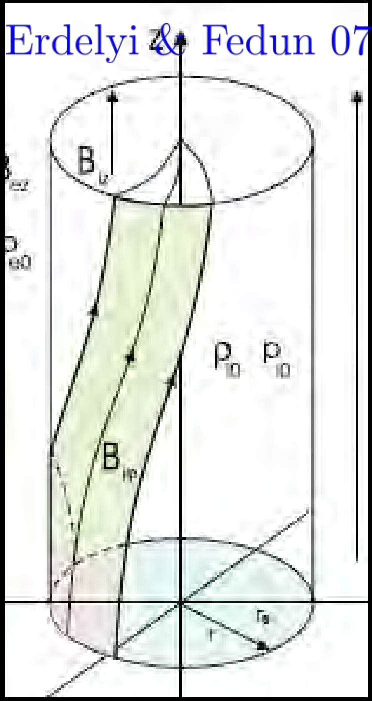
$$k_{c,l}R = \frac{d_l}{\sqrt{\rho_i/\rho_e - 1}}$$

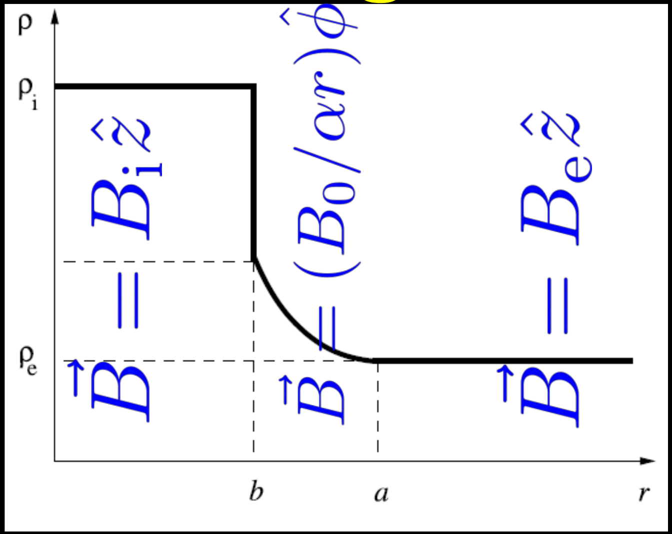
$$\rho(r) = \rho_e + (\rho_i - \rho_e) f(r)$$

$$f(r) = \begin{cases} 1, & 0 \le r \le R \\ (r/R)^{-\mu}, & r \ge R. \end{cases}$$

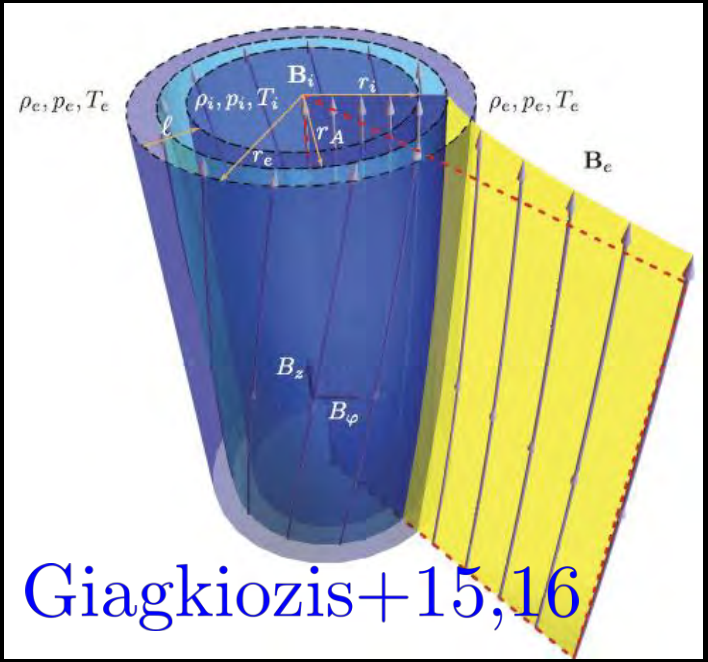
- cutoff axial wavenumber exists only when external density falls off sufficiently rapidly.
- All modes are trapped for all *k* when mu < 2. Hence for thin “loops”, period of standing sausage modes ~ kink modes ~ axial Alfven time. Their applications to QPPs not restricted to rapid ones (Lopin & Nagorny 15, 19).

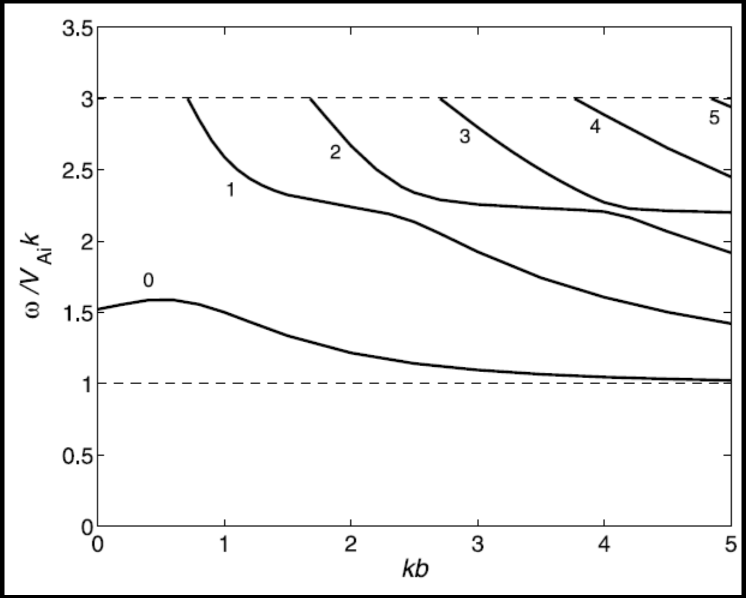
Magnetic twist





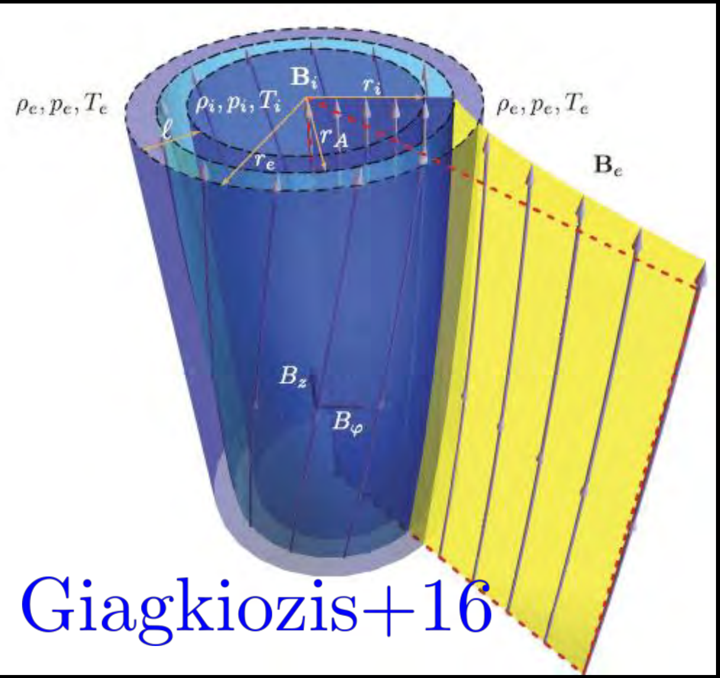
Khongorova+12

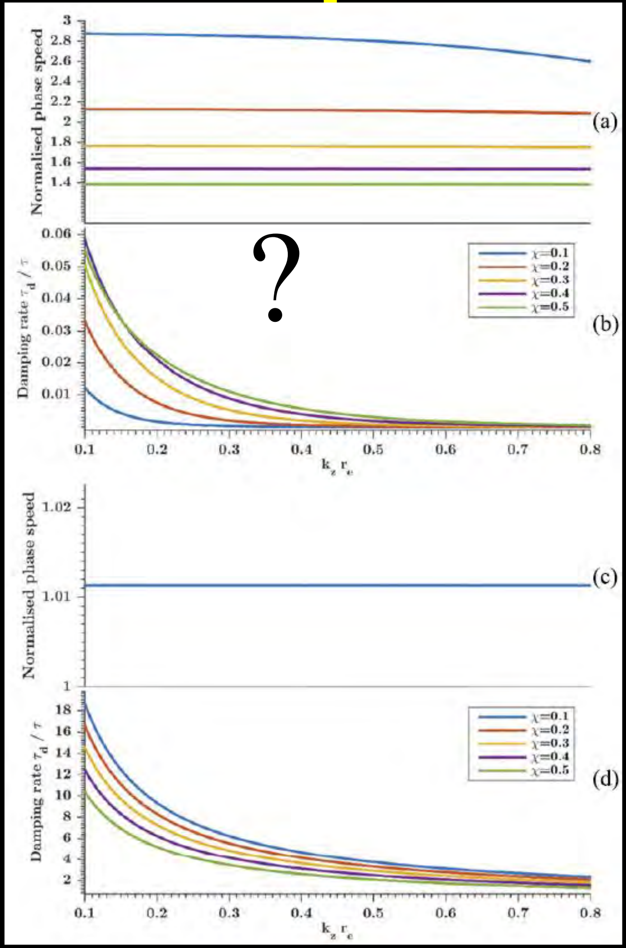




- In general, twist couples fast & m=0 Alfven.
- transverse fundamental trapped for all *k* when some external twist exists. Resonant absorption in Alfven continuum?

twist: resonant absorption in Alfven



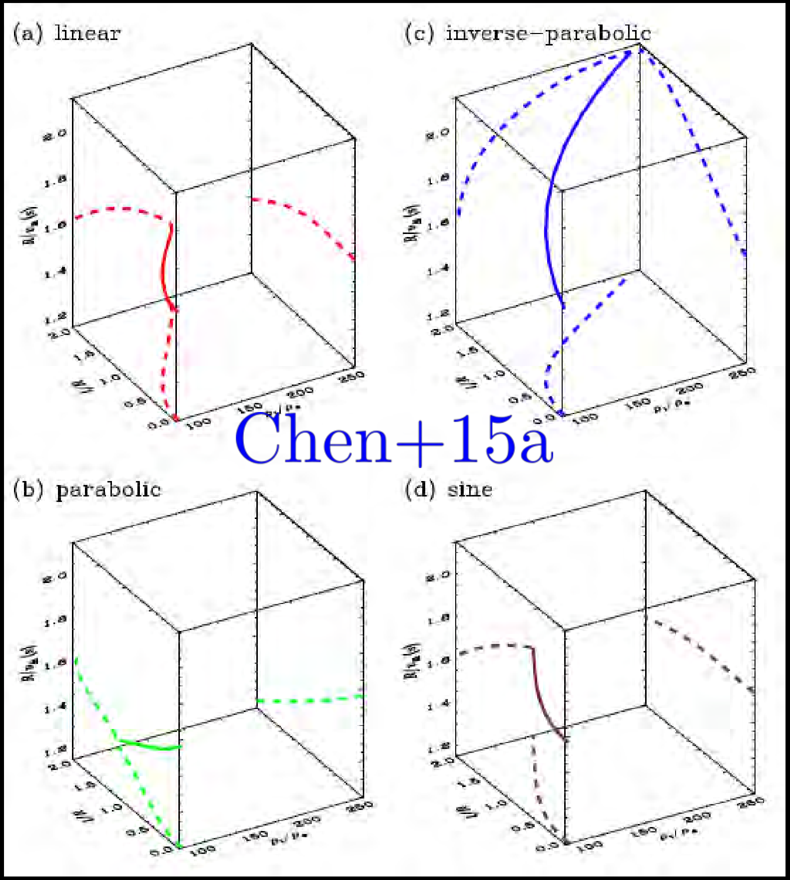


- A mode (vph ~ v_Ai) with decent observability, an extremely heavily damped mode with higher vph

Seismology

- In principle, all eigenmode analyses can be put to seismological use, after physical relevance established
- Standing modes
  - Individual fast sausage : inversion problem usually under-determined (Chen+15a …)
  - Multiple fast sausage
    - ✓ Periods alone: period ratios between fundamental harmonics help diagnose axial inhomogeneity in B (Pascoe+09), but not sure for that in density (implied in Cally & Xiong 18). Not explored in detail
    - ✓ Periods & damping times: not explored yet.
  - fast + slow sausage -> periods helpful for diagnosing plasma beta (Van Doorsselaere+11)
  - fast sausage + fast kink: Chen+15a, Guo+16, Roberts+19

Uncertainties

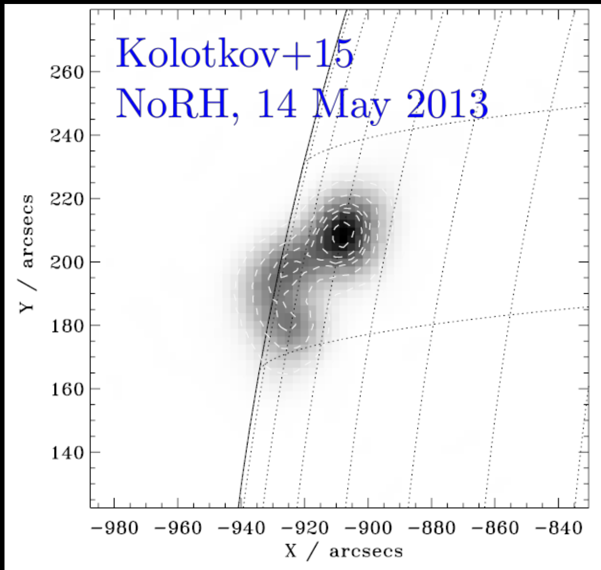


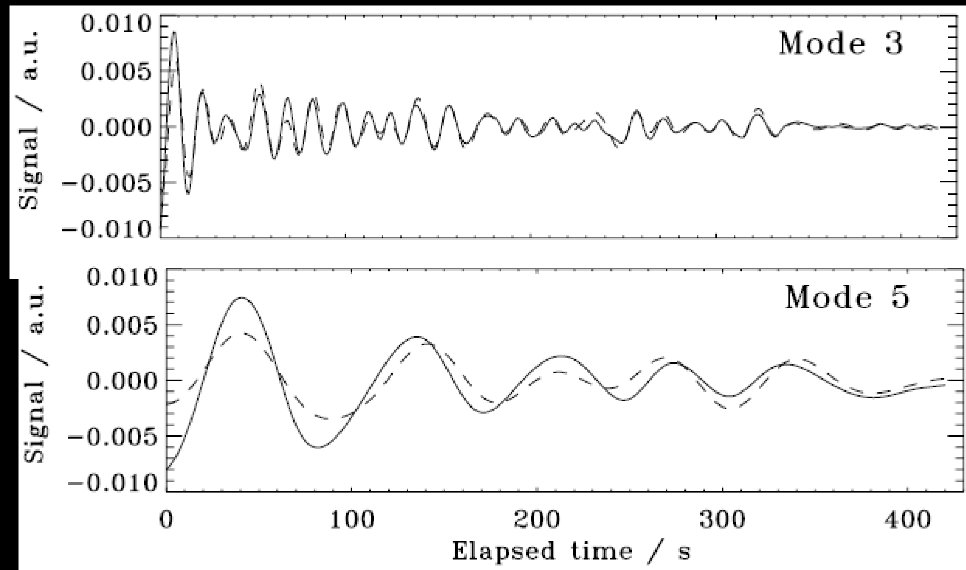
$$P_{\rm saus} = \frac{R}{v_{\rm Ai}} F_{\rm saus}\left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\rm i}}{\rho_{\rm e}}\right),$$

$$\frac{\tau_{\rm saus}}{P_{\rm saus}} = G_{\rm saus}\left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\rm i}}{\rho_{\rm e}}\right).$$

- *A tiny step forward from Edwin & Roberts: density trans. conti*
- $R/v_{\rm Ai}$: max/min = 1.8; den. contrast: max/min = 2.9, l/R: not constrained
- detailed functional form of density variation matters

the more measurements, the better (Guo+16)



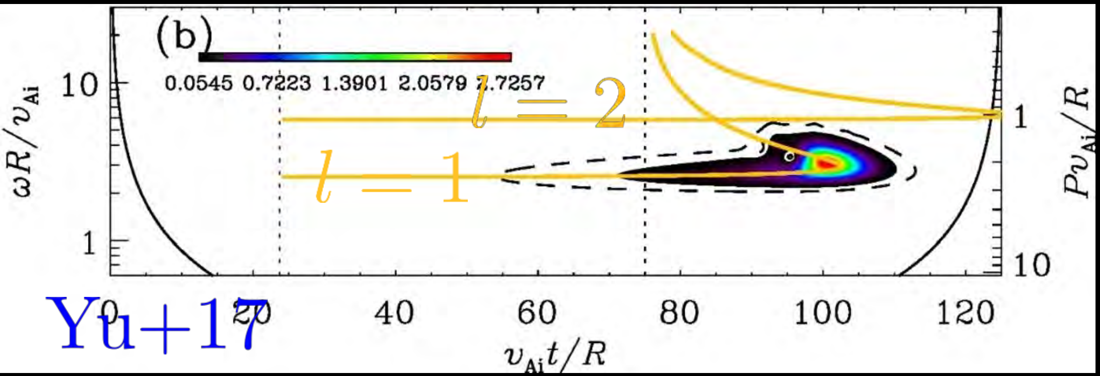


- geometric parameters known
- long period → fast kink damped by resonant absorption, short period → fast saus experiencing leakage

profile	l/R	$\rho_{\rm i}/\rho_{\rm e}$	$v_{\rm Ai}$ (km/s)	$P_{\rm kink,theory}$ (s)
linear	0.167	28.5	653.8	91.5
parabolic	0.240	28.4	657.7	89.2
inverse-parabolic	0.277	31.1	593.7	102.5
sine	0.284	29.9	620.5	95.9

Wavetrains from localized impulsive drivers

- Seismological application pointed out by Roberts+83, 84
- Wavelet analysis a proper tool
  - slabs: linear (Nakariakov+04, Pascoe+13AA, 14AA, Li+18, Goddard+19)
  - tubes: linear (Shestov+15, Yu+16,17) nonlinear (Pascoe+17)
  - nonetheless, in addition to equilibrium parameters, details of wavelets also depend on spatial & temporal extent of driver.
  - something irrelevant of the driver



If Morelets can be better localized in frequency, then the entire ridge can be used. Modern techniques (e.g., WSST) suitable for this purpose

$$\omega - h/v_{\rm gr}$$

Freq - wavepacket arrival time

Sausage waves in the lower solar atmosphere

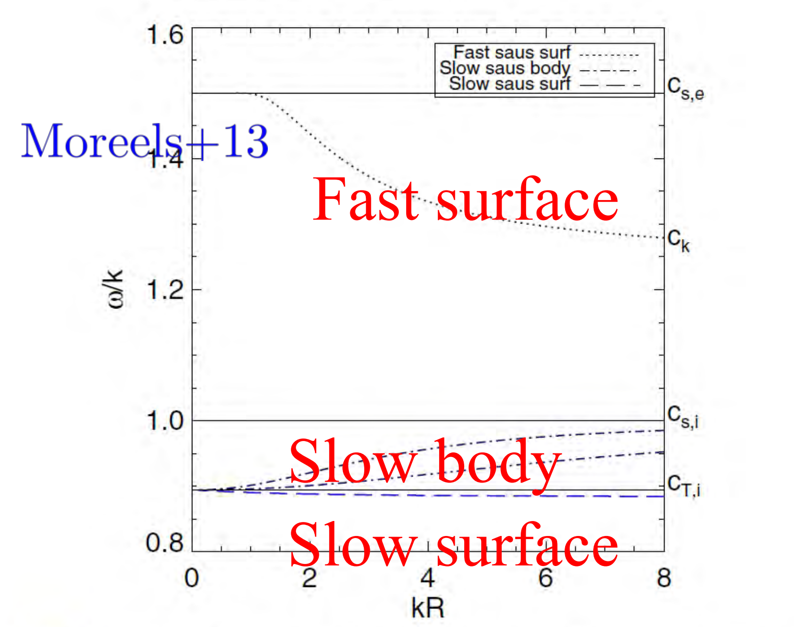


Fig. 2. Phase speed diagram of wave modes under photospheric conditions. We have taken $c_{A,i} = 2c_{s,i}$, $c_{A,e} = 0.5c_{s,i}$, and $c_{s,e} = 1.5c_{s,i}$. The

Table 1. Phase differences between the cross-sectional area variation and the intensity perturbation for different sausage wave modes.

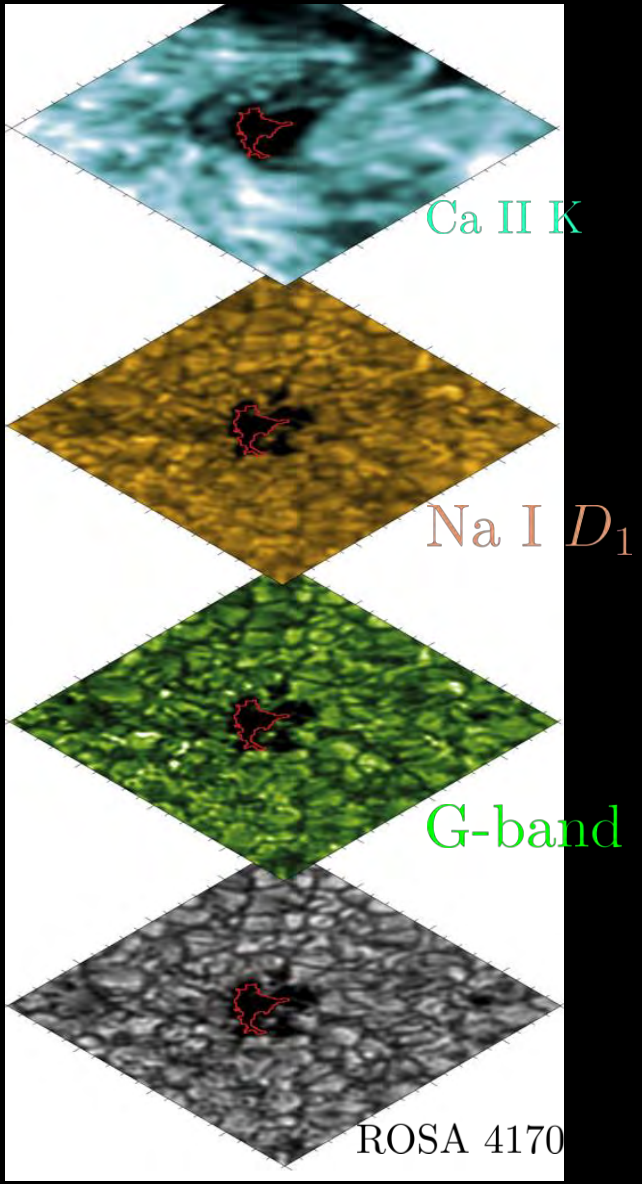
Wave mode	sign of $\mathcal{I}_1$	Sign of $\mathcal{I}_2$	sign of S_1	Phase behaviour
Slow surface	−	−	−	in phase
Slow body	+	+	+	in phase
Fast surface	+	−	−	in antiphase

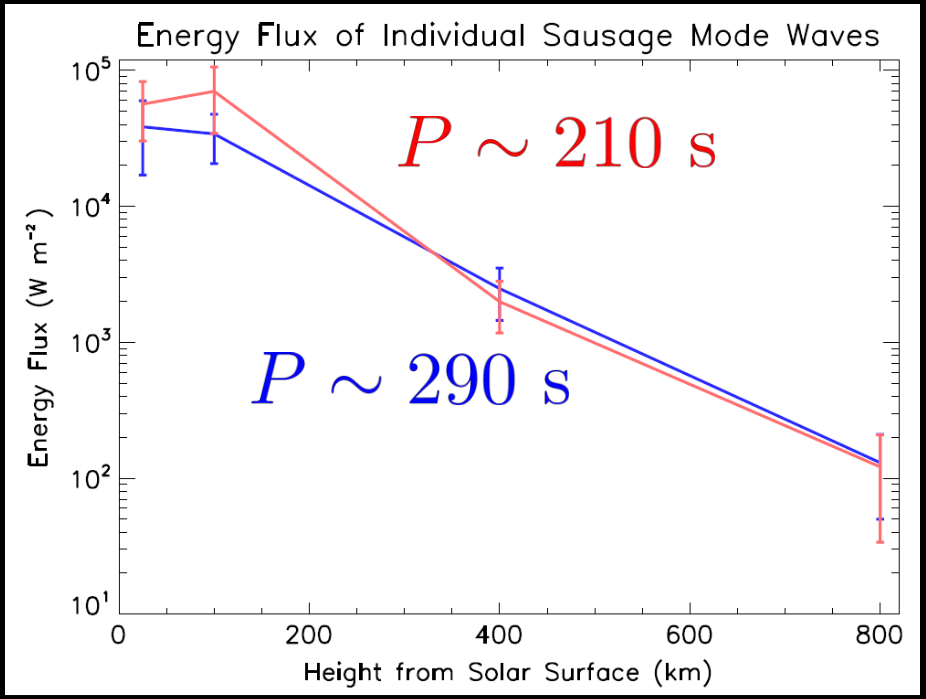
- Photospheric structures
  - Observational signatures and hence a seismological toolkit (Moreels & Van Doosselaere 13, Moreels+13) , with spectropolarimetric measurements in mind (also Fujimura & Tsuneta 09)
  - equilibrium model essential ER83, no gravity
  - emitting source supposed to be in LTE
- Chromospheric fibrils (Morton+12, Jafarzadeh+17), primarily based on recognizing variations in cross-sectional area

Sausage modes in photospheric structures

structure	Instrument/pass band	periodicities	Method	modes	Ref
pore in sunspot 7519	Swedish ST	20-70min (area only)	Wavelet	slow	Dorotovic+08
sunspot+pore	SST+Dunn ST	4-65min (area+intensity)	wavelet+ EMD	fast & slow standing(P ratio)	Dorotovic+14
pore	ROSA/DST	30-450s (area, int, anti-phase)	wavelet+ EMD	standing (P ratio)	Morton+11
pore	CRISP/SST	1.5,2, 3, 6.5min (area, int, in-phase)	Wavelet	standing	Moreels+15
pore	DST, multi-lambda	181-412s (area, int)	wavelet+ Fourier	prop. damping.	Grant+15
pore	DST	3-20mins (area, int)	wavelet+ EMD	standing, slow	Freij+16
pore	ROSA	2-12mHz (area, int)	wavelet+ EMD	body(up to 11mHz) surf(<=10mHz)	Keys+18

DST measurements of heavily damped upward-propagating modes [Grant+15]





- The apparent damping length may reach a fraction of the axial wavelength!
- This heavily damped slow surface sausage mode (SSSM) suggested to account for chromospheric heating above this pore

Energy carrying capabilities

$$\langle\overline{\mathrm{KE}}\rangle = C_r(\kappa_i R)^2\left(I_1(\kappa_i R)^2 - I_0(\kappa_i R)\,I_2(\kappa_i R)\right) + C_z(kR)^2\left(I_0(\kappa_i R)^2 - I_1(\kappa_i R)^2\right),$$

$$\langle\overline{\mathrm{ME}}\rangle = \frac{\omega_{\mathrm{A,i}}^2}{\omega^2}C_r(\kappa_i R)^2\left(I_1(\kappa_i R)^2 - I_0(\kappa_i R)\,I_2(\kappa_i R)\right) + \frac{\omega_{\mathrm{A,i}}^2}{\omega_{\mathrm{s,i}}^2}\frac{\left(\omega_{\mathrm{s,i}}^2 - \omega^2\right)^2}{\omega^2\omega_{\mathrm{s,i}}^2}C_z(kR)^2\left(I_0(\kappa_i R)^2 - I_1(\kappa_i R)^2\right)$$

$$\langle\overline{\mathrm{IE}}\rangle = \frac{\omega^2}{\omega_{\mathrm{s,i}}^2}C_z(kR)^2\left(I_0(\kappa_i R)^2 - I_1(\kappa_i R)^2\right),$$

$$\langle\overline{\mathrm{TE}}\rangle = \langle\overline{\mathrm{KE}}\rangle + \langle\overline{\mathrm{ME}}\rangle + \langle\overline{\mathrm{IE}}\rangle,$$

$$\langle\overline{\boldsymbol{S}}\rangle = 2\frac{c_{\mathrm{A,i}}^2}{\omega/k}C_r(\kappa_i R)^2\left(I_1(\kappa_i R)^2 - I_0(\kappa_i R)\,I_2(\kappa_i R)\right)\mathbf{1}_z,$$

$$\langle\overline{\boldsymbol{T}}\rangle = 2\frac{\omega}{k}C_z(kR)^2\left(I_0(\kappa_i R)^2 - I_1(\kappa_i R)^2\right)\mathbf{1}_z,$$

$$\langle\overline{\boldsymbol{F}}\rangle = \langle\overline{\boldsymbol{S}}\rangle + \langle\overline{\boldsymbol{T}}\rangle,$$

Moreels+15

$$\langle\overline{\mathrm{KE}}\rangle = \frac{\rho_{0,\mathrm{i}}}{4}\omega_{\mathrm{T,i}}^2\pi R^2\Xi_z'^2,$$

$$\langle\overline{\mathrm{ME}}\rangle = \frac{\rho_{0,\mathrm{i}}}{4}\omega_{\mathrm{T,i}}^2\pi R^2\Xi_z'^2\frac{\gamma}{2}\beta,$$

$$\langle\overline{\mathrm{IE}}\rangle = \frac{\rho_{0,\mathrm{i}}}{4}\omega_{\mathrm{T,i}}^2\pi R^2\Xi_z'^2\left(1 - \frac{\gamma}{2}\beta\right)$$

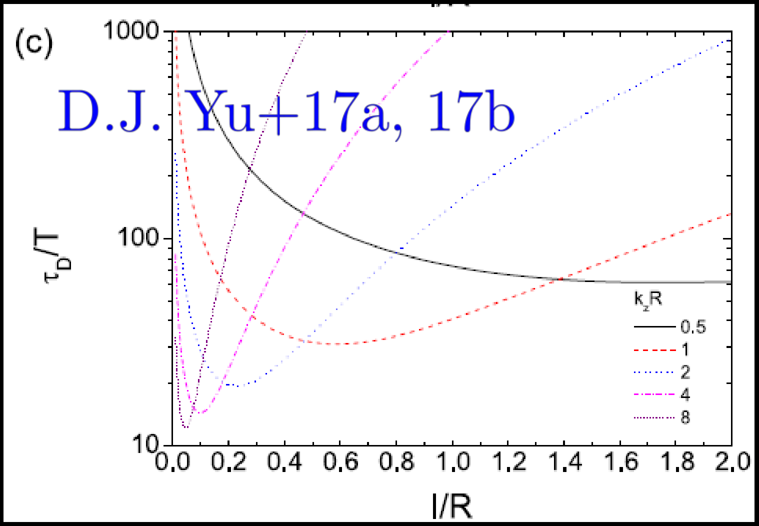
$$\langle\overline{\boldsymbol{S}}\rangle = \mathbf{0},$$

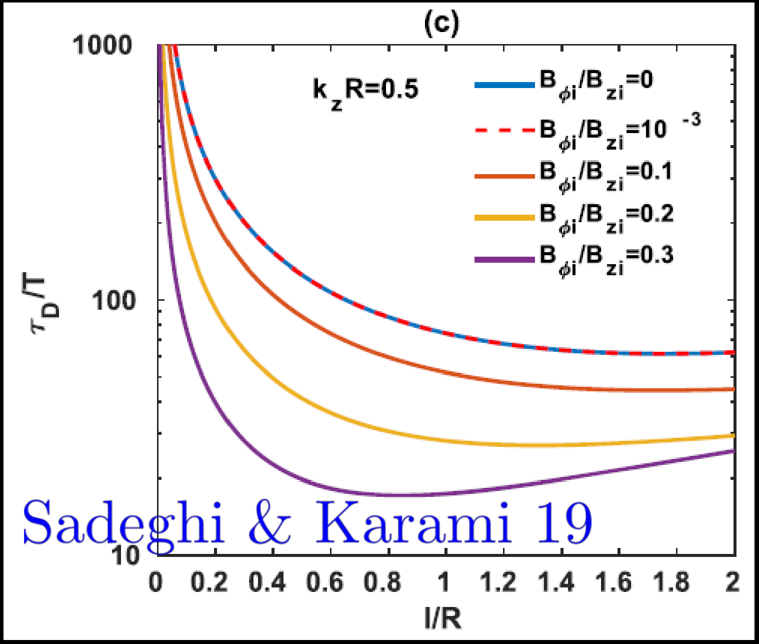
$$\langle\overline{\boldsymbol{T}}\rangle = 2\frac{\rho_{0,\mathrm{i}}}{4}\omega_{\mathrm{T,i}}^2\pi R^2\Xi_z'^2 c_{\mathrm{T,i}}\mathbf{1}_z,$$

$$\boldsymbol{v}_{\mathrm{g}} = c_{\mathrm{s,i}}\left(1 + \frac{\gamma}{2}\beta\right)\mathbf{1}_z.$$

- expressions for energy & energy flux densities available for both fast & slow
- SSSM pretty much confined to a photospheric structure (e.g., pore)

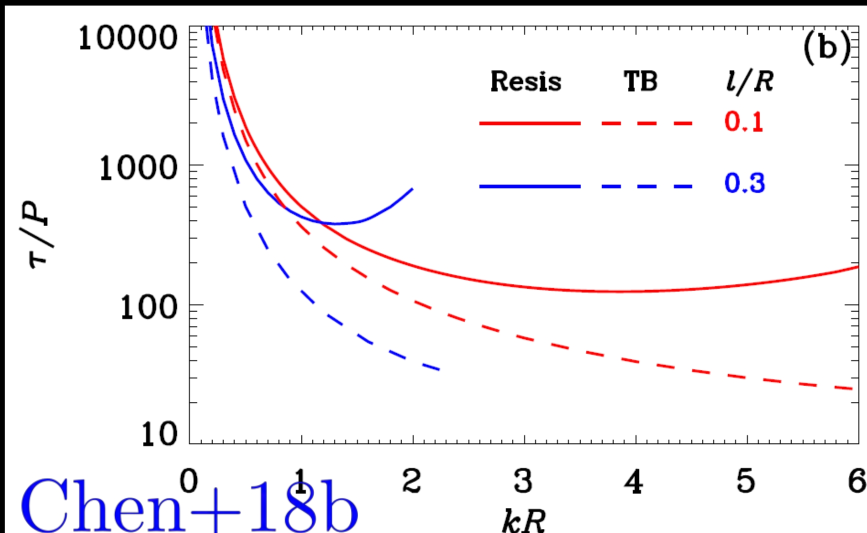
Damping: ideal (R.A. in cusp cont.)

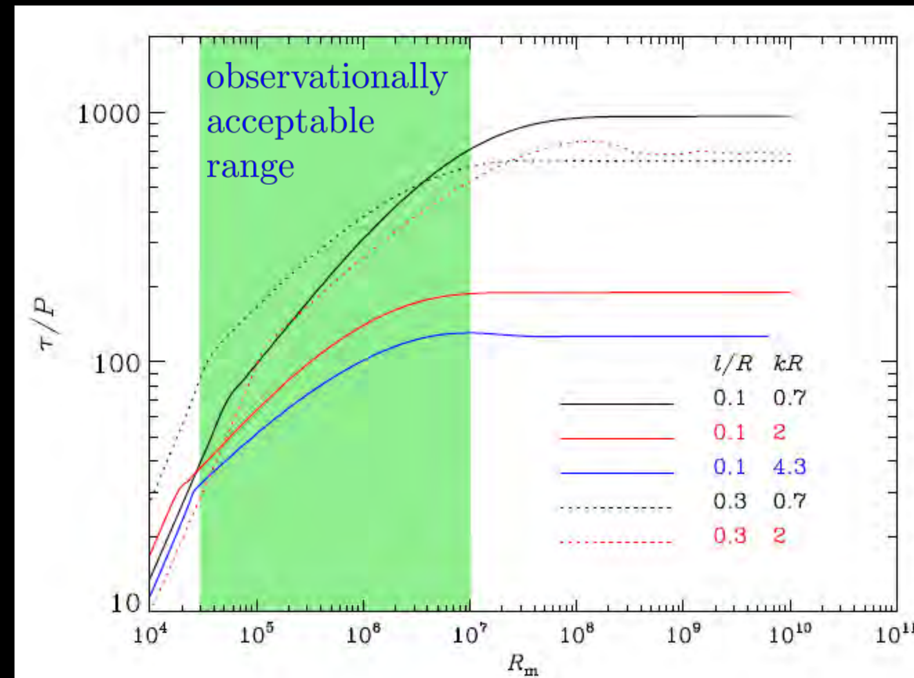




- continuous transverse variation of cusp frequency results in resonant coupling of an SSSM to cusp continuum (D.J. Yu+17a, 17b, 19)
- Damping efficiency enhanced when there is internal twist (Sadeghi & Karami 19)
- Notes: 1. Additional mechanism required for dissipating localized slow modes; 2. both studies use semi-analytic, thin-boundary (TB) framework (see review by Goossens+11)

Damping: non-ideal





- R.A. rate from resistive computations converge to thin-boundary (TB) results only when kR is small
- in photospheric structures, Ohmic resistivity from e-neutral collisions tends to be more efficient than R.A. (Chen+18b, Geeraerts+19?)

Summary

- coronal sausage waves
  - primarily (exclusively?) examined in the seismological context
  - improvements upon ER83 lead to
    - ✓ reconsideration of existence of cutoff axial wavenumbers, hence applicability of sausage modes to more than just "rapid" QPPs
    - ✓ resonant absorption as alternative to leakage for damping
  - forward modeling better done case-by-case
  - generation mechanisms not settled
- sausage waves in the lower solar atmosphere
  - growing interest from both seismological and heating standpoints, theories lag behind (gravity necessary but seldom addressed, Roberts 19, Pardi+14, …)
  - forward modeling in infancy but necessary (radiative transfer challenging)
  - heating efficiency not settled
- More to come after hearing your feedback